Numerical simulation studies of the convective instability onset in a supercritical fluid

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Numerical simulation studies in 2D with the addition of noise are reported for the convection of a supercritical fluid, ^{3}He, in a Rayleigh-Bénard cell. The noise addition is to accelerate the instability growth after starting the heat flow across the fluid, so as to bring simulations into better agreement with published experimental observations. Homogeneous temperature noise and spatial longitudinal periodic temperature variations in either top or bottom plates were programmed into the simulations. The second method was the most effective in speeding up the instability onset. For a small amplitude of the longitudinal perturbations, a semiquantitative agreement with the observations was obtained. The results are discussed in relation to predictions by El Khouri and Carlès.

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In recent papers, convection experiments of supercritical ^{3}He in a Rayleigh-Bénard cell with a constant heat current q were reported [1,2]. After q is started, the temperature drop $\Delta T(t)$ across this highly compressible fluid layer increases from zero, an evolution accelerated by the "Piston Effect" (PE) [3,4]. Assuming that q is larger than a critical heat flux necessary to produce fluid instability, $\Delta T(t)$ passes over a maximum at the time $t=t_p$, which indicates that the fluid is convecting and that plumes have reached the top plate. Then truncated or damped oscillations, the latter with a period t_{osc}, are observed under certain conditions before steady-state conditions for convection are reached, as described in Refs. [1,2]. The scenario of the damped oscillations, and the role of the PE has been described in detail in Refs. [5] and [6] and will not be repeated here. The height of the layer in the RB cell was $L=0.106$ cm and the aspect ratio $\Gamma=57$. The ^{3}He convection experiments along the critical isochore extended over a range of reduced temperatures between $5\times10^{-4}\leqslant\epsilon\leqslant0.2$, where $\epsilon=(T-T_c)/T_c$ with $T_c=3.318$ K, the critical temperature.

The scaled representation of the characteristic times t_{osc} and t_p versus the Rayleigh number, and the comparison with the results from simulations in 2D has been described in Ref. [7]. Good agreement for the period t_{osc} was reported. However a systematic discrepancy for the times t_p shows that in the simulations the development of convection is retarded compared to the experiments. This effect increases with decreasing values of $[\mathrm{Ra}^{\mathrm{corr}}-\mathrm{Ra}_c]$, where $\mathrm{Ra}^{\mathrm{corr}}$ is the Rayleigh number corrected for the adiabatic temperature gradient as defined in Refs. [1,2] and Ra_c is the critical Rayleigh number for the experimental conditions, 1708. This numerical value is the predicted one for a fluid layer contained between two isothermal plates. Because of the high thermal conductivity of the copper plates, any temperature inhomogeneities due to the heating element on the bottom plate is smoothed out and hence $\mathrm{Ra}_c=1708$ is consistent with the observed value (see Fig. 4 of Ref. [1]). With this value of Ra_c, the simulations, in which isothermal plates were used, are also consistent with observations as shown for the steady-state conditions (see Fig. 4 of Ref. [9]).

This discrepancy in t_p is shown in Fig. 1 of Ref. [7], in particular in Fig. 1(b) for $\epsilon=0.2$ and $q=2.16\times10^{-7}$ W/cm^2 ($[\mathrm{Ra}^{\mathrm{corr}}-\mathrm{Ra}_c]=635$), where an experimental run is compared with simulations for the same parameters. Here, clearly the profile $\Delta T(t)$ from the simulations shows a smooth rise until the steady-state value, $\Delta T=qL/\lambda=125\ \mu$K has been reached, where λ is the thermal conductivity. Only at $t\approx90$ s does convection develop, as shown by a sudden decrease of $\Delta T(t)$. By contrast, the experimental profile shows a much earlier development of convection. Figure 1 of Ref. [7] is representative of the observations at low values of $[\mathrm{Ra}^{\mathrm{corr}}-\mathrm{Ra}_c]$. At high values, both experiment and simulations show the convection development to take place at comparable times, as indicated in Fig. 5(b) of Ref. [7], and specifically in Fig. 2(a) of Ref. [6], where $[\mathrm{Ra}^{\mathrm{corr}}-\mathrm{Ra}_c]=4.1\times10^5$. It is the purpose of this report to investigate the origin of this discrepancy by further simulation studies.

El Khouri and Carlès [8] studied theoretically the stability limit of a supercritical fluid in a RB cell, when subjected to a heat current q started at the time $t=0$. Their fluid was also ^{3}He at the critical density, and the same parameters as in Ref. [1] were used. They calculated the time t_{instab} and also the corresponding $\Delta T(t_{\mathrm{instab}})$ for the onset of fluid instability and they determined the modes and the wave vectors of the perturbations for different scenarios of q and ϵ. For $t>t_{\mathrm{instab}}$ inhomogeneities in the RB cell and noise within the fluid will produce perturbations which will grow, from which the convection will develop. An indication of the growth of convection is a deviation of the $\Delta T(t)$ profile in the experiments or in the simulations from the calculated curve for the stable fluid [see, for instance, Eq. (3.3) of Ref. [5]]. It is readily seen from simulation profiles such as Figs. 1(a) and 1(b) in Ref. [5] that the deviation becomes significant for t only slightly below t_p—the maximum of $\Delta T(t)$. In simulations, the effective start of convection can also be seen from snapshots in 2D of the fluid temperature contour lines at various times, as shown in Fig. 5 of Ref. [9].

Carlès [8] has argued that the reason for the discrepancy for the time t_p between experiment and simulation is that in the former, the physical system has noise and inhomogene-

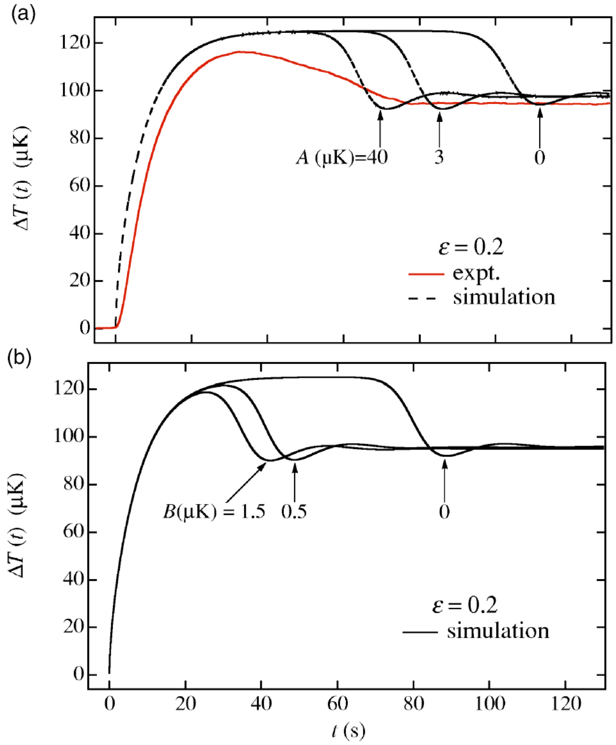

FIG. 1. The temperature profile $\Delta T(t)$ at $\epsilon=0.2$, $q=2.16\times10^{-7}$ W/cm^2. (a) From experiments (solid line with noise) and from several simulations (dashed lines) with $\Gamma=5.1$ and uniform temperature noise on the top plate with amplitude A. (b) From simulations with $\Gamma=8$ and with time independent, horizontal temperature variations with period $2L$ and amplitude B, imposed along either the top plate or the bottom plate.

ities which cause the perturbations beyond t_{instab} to grow into the developed convection. By contrast simulations have a much smaller noise. Therefore in the simulations the perturbations take a longer time to grow than in the physical system, leading to a larger t_p than observed. Carlès' comment led us to try as a first step imposing a thermal random noise on the top plate of the RB cell, which was to simulate fluctuations in the upper plate temperature control of the laboratory experiment. The temperature of the plate was assumed to be uniform, because of the large thermal diffusivity $D_T \approx 2\times10^4$ cm^2/s of the copper plate in the experiments. Accordingly simulations in 2D were carried out by the numerical method described in Ref. [5] with a homogeneous time-dependent temperature random fluctuation of given rms amplitude imposed on the upper plate. This implementation consisted of adding or subtracting random temperature spikes T_t at time t with a programmed rms amplitude at steps separated by 0.02 s. This interval is much larger than the estimated relaxation time of the top plate over a distance $2L$, approximately the wavelength of the convection roll pair. Values of the variance $A=\sqrt{\langle(T_t-\langle T_t\rangle)^2\rangle}$ were chosen between 0 and 40 μK. The range of the A values was taken well beyond the estimated fluctuation rms amplitude during the experiments [1] of ≈ 1 μK/$\sqrt{\text{Hz}}$. Three representative curves with 0, 3, and 40 μK are shown in Fig. 1(a) by dashed lines for $\epsilon=0.2$ for $q=2.16\times10^{-7}$ W/cm^2, $L=0.106$ cm and $\Gamma=5.1$. For this value of q, the calculation by El Khouri and Carlès [8] give $t_{\text{instab}}=6.3$ s and $\Delta T(t_{\text{instab}})=75$ μK. In the simulation without imposed noise, the start of convection has therefore been considerably delayed relative to t_{instab}. The injection of random noise has a significant effect in developing convection at an earlier time, even for $A \ll \Delta T$ (steady state) ≈ 125 μK. In Fig. 1(a) the three curves are also compared with the experimental one, shown by a solid line. Here we have not incorporated into the simulations the delay affecting the experimental temperature recording, so that they could be intercompared more readily, and also with predictions [8]. However this operation will be presented in Fig. 3. Further simulations with added random noise were carried out for $\epsilon=0.2$ and 0.05 where the $\Delta T(t)$ time profiles are not shown here. Finally we mention that no difference in the recorded profile $\Delta T(t)$ was found when noise with the same rms amplitude $A=5$ and 10 μK was injected into the bottom plate (instead of the top plate) for simulations at $\epsilon=0.05$.

Figure 2(a) shows a plot of the time of the developed convection, represented by t_p, versus the random rms amplitude A for three series of simulations, all taken for a cell with $\Gamma=5.1$. They are (a) and (b) $\epsilon=0.2$, $q=2.16$ and 3.89×10^{-7} W/cm^2, and (c) $\epsilon=0.05$, $q=60$ nW/cm^2 ($[\text{Ra}^{\text{corr}}-\text{Ra}_c]=635$, 1740, and 4200). The simulation results, shown by solid circles, are compared with the experimentally observed t_p shown by horizontally dotted-dashed lines. It can be clearly seen that noise imposition, which creates a vertical disturbance across the fluid layer, reduces the time of convection development. While the decrease of t_p is strong for small values of A, it saturates at a certain level of noise amplitude. The gap between simulations and experiment increases with a decrease of $[\text{Ra}^{\text{corr}}-\text{Ra}_c]$, namely as the fluid stability point is approached. A "critical slowing down" is seen in the effectiveness of the perturbations in triggering the instability. Hence this mode of noise introduction fails, because its amplitude is limited to the vertical z direction and it evidently couples only weakly into the convective motion.

In parallel with the present experiments, Amiroudine also carried out a systematic study of simulations on supercritical ^{3}He in a RB cell for several values of ϵ and q. He used a numerical scheme based on the exact Navier-Stokes equation as described in Ref. [6]. The resulting profiles $\Delta T(t)$ could be compared with those from experiments done under nearly the same conditions. In his simulations, homogeneous temperature random noise was again imposed on the top plate reported above, and at zero noise, the t_p values tended to be somewhat smaller than in the results of Fig. 2(a).

Here we mention that the onset of convection in the simulations is further influenced by the aspect ratio Γ. The simulations described above, but without noise, were carried out in a cell $\Gamma=5.1$ having periodic lateral boundaries. Further simulations with zero noise for $\epsilon=0.2$ with $\Gamma=8.0$, 10.2, 20.5, and 41.0 were carried out, and showed a decrease of the convection development time from ≈ 90 s, tending to a constant value of ≈ 60 s above $\Gamma=20$. This shift in the onset of instability is due to the decreased finite size effect which the rising plumes experience with increasing Γ, in spite of the periodic boundary conditions. The effect of a smaller

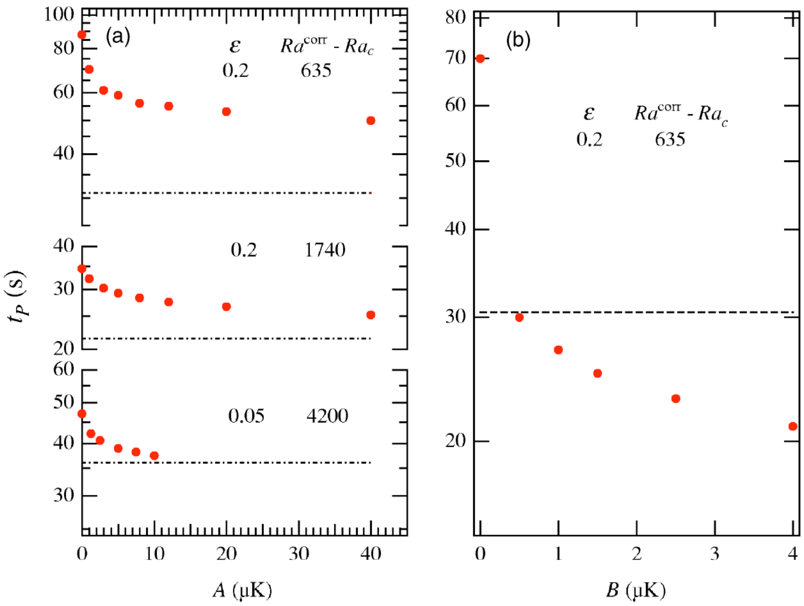

FIG. 2. (a) The time for effective development of convection, characterized by t_p versus A (homogeneous temperature noise imposed on the top plate). The horizontal dotted-dashed lines indicate the observed t_p, corrected for instrumental recording delay. (b) The time for effective development of the convection, labeled by t_p versus B (time-independent periodic temperature variations along the top or bottom plate). The horizontal dashed line indicates t_p as obtained by experiment, corrected as before.

aspect ratio is then (as observed) to delay the onset, and that this stability increase is due to the quantization of admissible perturbations in the cell. This can be seen by comparing the curves labeled "O" in Figs. 1(a) and 1(b) with $\Gamma=5$ and 8, respectively.

The next step in our attempts, stimulated by communications with Carlès, was introducing perturbations into the simulations via some time-independent nonhomogeneous temperature variation proportional to $\sin(2\pi x/P)$ where x is the horizontal coordinate and P is the period. We first introduced a temperature variation along the top plate with an amplitude B (in μK) and period $P=2L$, nearly the same as the wavelength of a pair of convection rolls. The temperature of the bottom plate was kept homogeneous. This "Gedanken Experiment" implies that the material of the top plate permitted a temperature inhomogeneity, which of course is not realized in the experiment. However a small temperature excursion along x in the top plate can trigger the same kind of nonhomogeneous perturbations as those which, in the real experiment, provoke the onset of convection. One possible origin of such perturbations, besides thermal noise, could be the roughness of the plates or their slight deviation from parallelism. Such geometrical defects could of course not be implemented in the numerical simulations with the mesh size used, which is why we elected to force a small temperature perturbation instead, with similar effects on the onset. As a control experiment, we also made a simulation with $P=L$.

Figure 1(b) shows representative profiles $\Delta T(t)$ for the parameters $\epsilon=0.2$ and $q=2.16\times10^{-7}$ W/cm^2 and with $B=0$, 0.5, and 1.5 μK, and for $\Gamma=8$. As B is increased from zero, there is a large decrease in the time for convection development, represented by t_p, which is plotted versus B in Fig. 2(b). The horizontal dashed line shows the t_p from the experiment, and this plot is to be compared with Fig. 2(a). For an inhomogeneity amplitude of only $B=0.5$ μK, t_p is nearly the same for simulations and experiment. This indicates that this very small imposed noise amplitude (as compared with ΔT in the steady state) in the simulations might be comparable to the natural noise amplitude in the physical system. Higher imposed amplitudes B in the simulations lead then to an onset of convection at earlier times than in the experimental system. Simulations were also carried out at $B=0$, 0.5, and 1.5 μK where the perturbations were applied to the bottom plate, and where the temperature of the top plate was kept uniform. There was no significant difference from the curves shown in Fig. 1(b). Hence again, the development of the convection seems independent of whether the perturbation is applied on the top or on the bottom plate, which is consistent with the experience with the homogeneous temperature noise. It is conceivable that more efficient ways in generating low amplitude noise in simulations will produce a still faster convection development.

By contrast, simulations with $B=2$ μK and $P=L$ (not presented here) show no difference from those with $B=0$. Hence the nucleation of the convection is accelerated if the period is in approximate resonance with the wavelength of a convection roll pair. The values of steady-state ΔT and t_{osc} are only marginally affected by the noise.

We note from Fig. 1(b) that the simulation curve calculated for $B=0$ shows the fluid not convecting until ≈ 70 s. For the curves with $B=0.5$ μK, the start of deviations from the stable fluid curve cannot be estimated well from Fig. 1(b) but is readily obtained from the data files, which tabulate $\Delta T(t)$ to within 1 nK. For $B=0.5$ μK, systematic deviations $\delta\Delta T(t,B)\equiv[\Delta T(t,B=0)-\Delta T(t,B)]$ increase rapidly from 1 nK for $t>8$ s (where $\Delta T\approx 85$ μK), a value comparable with the predicted $t_{instab}=6.3$ s, $\Delta T(t_{instab})=75$ μK [8]. However a comparison with predictions becomes more uncertain as B is increased and no longer negligible compared with the steady-state ΔT. Then it is expected that the base PE heat flow will become itself influenced by the perturbations. In that case the stability analysis in Ref. [8] becomes irrelevant, since the base flow, the stability of which is analyzed, has been significantly modified by the perturbations. We also note that the time interval $\delta t\equiv[t_p-t_{instab}]$ between the first sign of instability ($\delta\Delta T>0$) and t_p is ≈ 20 s, and roughly independent of B. This represents approximately the period taken by the convection to develop and for the plumes to reach the top plate boundary.

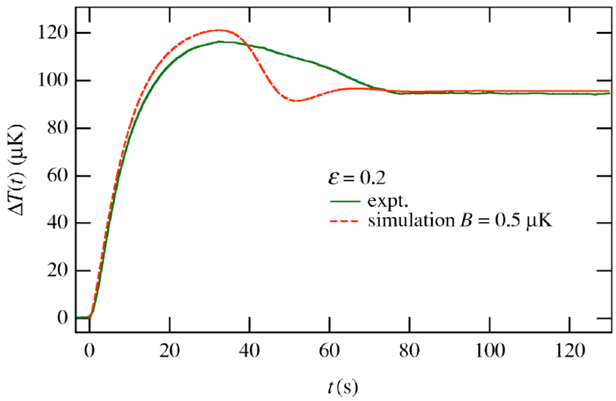

FIG. 3. Comparison of the profile $\Delta T(t)$ from experiment and from simulations with $B=0.5\ \mu$K. To make the comparison realistic, the simulations have been convoluted with the same "instrumental" delay time $\tau=1.3$ s which has influenced the shape of the experimental curve.

In Fig. 3 we show the profiles $\Delta T(t)$ from the experiment and from the simulations with a periodic perturbation amplitude $B=0.5\ \mu$K. This amplitude was chosen among those presented in Fig. 2(b) simply because it gave the best match between the experimental and the numerical values of t_p. For an optimal comparison, the delay affecting the experimental temperature recording was incorporated into the simulation curve. For this, the delay function with the instrumental time constant $\tau=1.3$ s [1] was folded into the simulation curve by a convolution method. This operation retards the initial rise of the temperature drop by the order of 2–3 s, and brings both experiment and simulations into fair agreement in the regime where the fluid is stable. The time t_p for the maximum is now closely the same for both experiments and simulations. However beyond the predicted instability time $t_{\text{instab}}=6.3$ s, the experimental curve starts to deviate more rapidly with time than the numerical simulations from the calculated curve for the fluid in the stable regime. As discussed previously [2], for these parameters of ϵ and q the experiment does not show damped oscillations, which are observed for higher values of q. In the steady state, the agreement is very good.

In a perfect RB cell with a very large aspect ratio without inhomogeneities in the horizontal plates and the side wall, the metastable nonconvecting state persists for a long time beyond the predicted convection onset after the heat current is started. In the RB cell used in experiments, inhomogeneities will cause perturbations which help developing the convection. The time t_p of the first peak in the $\Delta T(t)$ profile can be used as a measure of time where the convection is developing. The measured times t_p for supercritical ^{3}He at various values of ϵ and q, scaled by the appropriate diffusive relaxation time, were found to collapse on one curve as a function of the Rayleigh number [Fig. 5(a) of Ref. [7]]. Simulations with imposed periodic perturbations of a given amplitude $B=0.5\ \mu$K, as described above, were found to represent the convection onset in the experiments. An obvious extension is to conduct further simulations for the same parameters as used before and shown in Fig. 5(b) of Ref. [7], all with the same imposed perturbation amplitude $B=0.5\ \mu$K. It will then be interesting to see whether the scaled values of t_p versus the Rayleigh number will collapse on the same curve as the experimental data. This will be a step towards understanding the data shown in Fig. 5(a) of Ref. [7].

In conclusion, we have shown that including into the simulations a temperature perturbation with a favorable longitudinal periodicity into either the top or the bottom plate produces an earlier start in the convective instability, which for a small perturbation amplitude gives consistency with experimental observations. As the amplitude is further increased, the development of convection occurs earlier than the observed one. Here we have limited ourselves to an example at a low value of $[\text{Ra}^{\text{corr}}-\text{Ra}_c]$, where the delay has been particularly large with respect to the experiment.

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